The Space Elevator

Exeter Mathematics School

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Introduction

A Space Elevator is a hypothetical structure that would connect the surface of the Earth to Geospace via a tether. By creating a lift ascending from Ground Level to beyond the atmosphere we would eliminate the need for rocket propulsion, since cargo such as satellites could instead be elevated up to a station along the tether. A Space Elevator would be a much safer and less costly method of getting to Space than rocket combustion, possibly lowering the cost per kilo to between $10-£100.
The Elevator comprises of several structural features, the Base Station, the Tether, the Climbers and the Counterweight. Centrifugal forces would be strongest at the top of the tether and keep it upright and the Base Station would keep it attached to the ground. The Counterweight is necessary to make sure there the Centre of Mass of the system stays above Geostationary Altitude, thus maintaining a larger centrifugal force and keeping the cable taut.

Project Objectives

1. Research the various Physical subjects involving a Space Elevator
2. Investigate the viability of the Structure
3. Create a Program simulating the subjects and areas I have researched
4. Look into the required technological advancements and related theoretical structures

Base Location

The ideal global position of the Elevator base is at the equator. While it is possible for the Elevator to be built at larger latitudes, the ideal positioning would be two degrees around the equator. A non-equatorial Elevator would experience a large tension since forces of gravitation pull it towards the equatorial plane, and there would also be a larger distance required to get the centre of mass to GEO. Additionally, along the equator there is relatively little violent weather. Coriolis forces caused by the Earth's rotation mean that hurricanes and cyclones travel away from the Equator, making it an ideal area for a Base Station.
The Coriolis Effect would also impact the Elevator Climbers too. In a reference frame of anticlockwise rotation the coriolis force would act to the right. Since Earth rotates anticlockwise there would be a force on the Climber westward as it ascends. The relationship between the force and the velocity of the Climber means that it cannot travel too fast or the Coriolis forces will be too extreme for the Elevator to handle.

Velocity and Altitude

Since the cable is attached to the Earth it will rotate along with it. This causes it to act as a circular motion rather than regular satellite motion. An increase in altitude means an increase in velocity whereas for a satellite an increase in altitude means a decrease in orbital velocity. The equation on the left (v_1) models the velocity of a point on the Space Elevator's tether and the equation on the right (v_2) models the orbital velocity of a satellite at the same altitude.

$$v_1 = \omega r \quad (1)$$

$$v_2 = \sqrt[2]{\frac{GM}{r}} \quad (2)$$

ω is the rotational speed and is equal to that of the Earth's, G is the gravitational constant, M is the mass of Earth in kg, and both r's are the distance from the center of mass of the Earth. At Geostationary altitude the velocity provided by both the equations is the same (3070ms^{-1}), meaning that both a released projectile and a satellite would orbit in a circular Geostationary orbit. One of the major advantages of the Space Elevator would can be observed on the Left Graph: Past the point of Geostationary Altitude the Tether Speed increases proportionally, hence any released projectile at that height would assume that speed.
If a design of the Space Elevator in which the cable extends far into space instead of a counterweight is pursued then it would allow for the buildup of velocity. This could be used for orbital trajectories through the Solar System or into Deep Space, a disadvantage of this would be the lack of protection from our magnetosphere at such distances from the Earth. Approximately 53,100km altitude would be required for escape velocity from the Earth.

Apparent Gravitational Field

As a passenger goes rides up the Space Elevator there will be an apparent gravitational field due to the counteracting forces of Gravity and centrifugal force. We can derive and equation for the resultant field using Newton's Law of Gravitation, the equation for Centrifugal Force and Newton's Second Law, $F = ma$.

$$F_{grav} = \frac{GMm}{r^2} \quad (3)$$

$$F_{cent} = m\omega^2 r \quad (4)$$

$$F = ma \quad (5)$$

G = Gravitational Const, M = mass of planet, m = mass of object, in this case a climber, r = distance from the centre of the Earth, ω is the rotational speed of the Earth, a is acceleration

We use F=ma to derive both the centrifugal acceleration and gravitational acceleration, given by Equations 6 and 7.

$$a_{cent} = \omega^2 r \quad (6)$$

$$a_{grav} = -\frac{GM}{r^2} \quad (7)$$

The combination of these values gives the equation for the apparent gravitational field.

$$g = -\frac{GM}{r^2} + \omega^2 r \quad (8)$$

Van Allen Radiation Belts

One of the major threats to a Space Elevator would be the Van Allen belts. The Van Allen radiation belts are regions within Earth's magnetosphere consisting of high-energy protons and electrons trapped by Earth's magnetic field. They are a primary concern in regards to space travel because of the threat of radiation to humans and the effects of radiation on materials. There are two belts, an inner belt and an outer belt, however it has been shown that this amount is not fixed and three belts have been observed briefly before destruction.
The shapes of the two main belts has been found to be much more dynamic than previously thought, levels of different ions in certain regions changes due to solar winds. In addition, the axis of the belts is the Earth's magnetic poles rather than the geographic ones. This means the belts are slightly off equator by about 11.5 degrees.

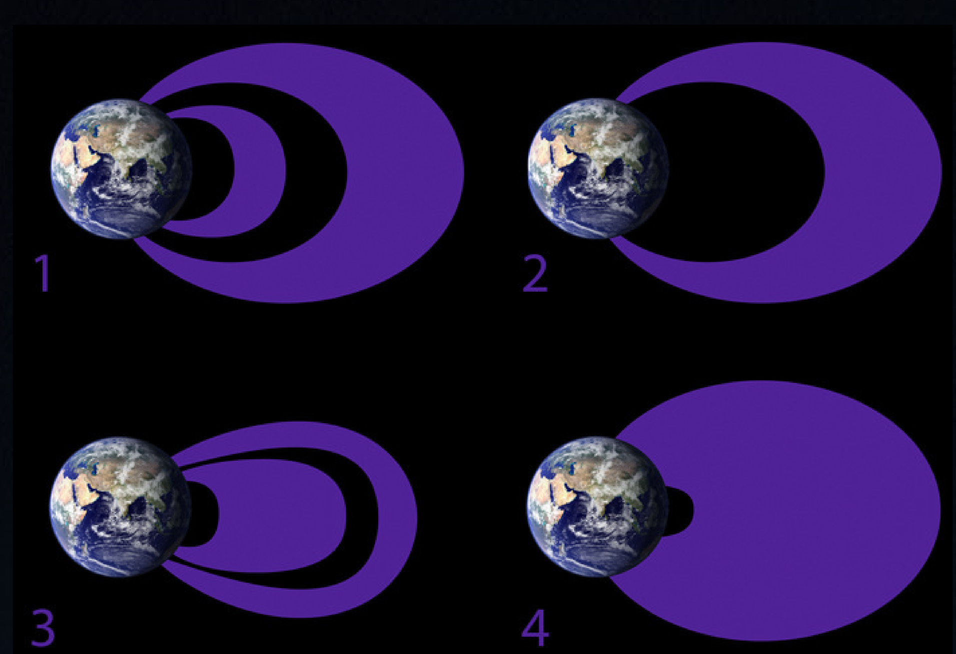

Figure 2: Van Allen Radiation Belts Composition, 1: Traditional View of the Belts, 2: High Energy Electron Map, 3: Low Energy Electron Map , 4: Electron Composition During Solar Storms

4mm of aluminium was enough to protect the astronauts during the Apollo missions, but the Climber would require a lot more protection for the passengers as it spends more time in the region. The radiological effects on materials considered for the Space Elevator is not entirely known. There is a chance of structural deformation from radiation via radiolysis of the structure.

Tether Requirements

The primary problem concerning the Space Elevator is the lack of suitable materials we have to make it. In order to carry its own weight the material would need to be low-density and have a tensile strength of approximately 50 Gigapascals. For comparison Steel has a tensile strength of 370 Megapascals, which is less than 1%.

0.0.1 Carbon Nanotubes

Theoretically, Carbon Nanotubes are the solution to this problem. Carbon Nanotubes are a layer of graphene rolled into a cylindrical lattice. Graphene is a single layer of carbon atoms each with sigma bonds to three neighbouring atoms. This causes Graphene to have an overall hexagonal structure. Carbon Nanotubes have an ideal theoretical tensile strength of 100 GigaPascals. There are two types of Carbon Nanotubes, Single-walled Carbon Nanotubes (SWCNTs/SWNTs) and Multi-walled Carbon Nanotubes (MWCNTs/MWNTs). Below is a table that compares the properties between the two.

Property	SWCNT	MWCNT
Relative Density	0.8g/cm^3	1.8g/cm^3
Elastic Modulus	1TPa	0.3-1TPa
Strength	50-500GPa	10-60GPa
Resitivity	5-50 $\mu\Omega$	5-50 $\mu\Omega$
Thermal Conductivity	$3000\text{Wm}^{-1}\text{K}^{-1}$	$3000\text{Wm}^{-1}\text{K}^{-1}$

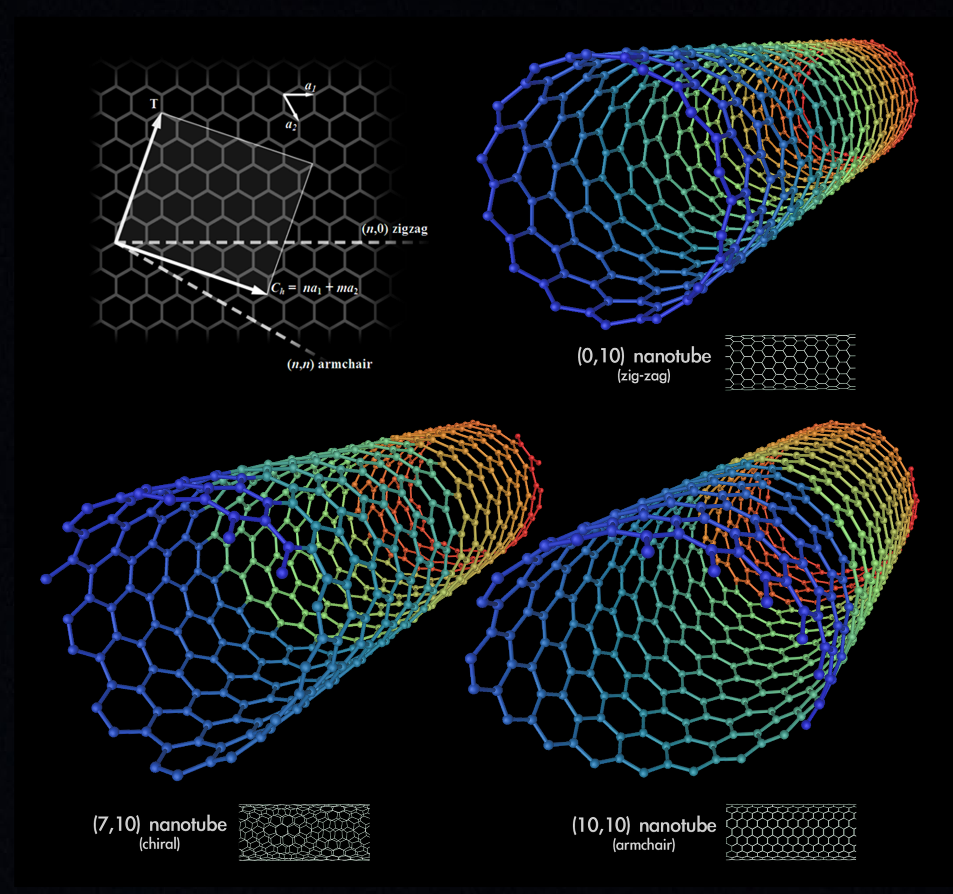

Figure 1: Different Types of Carbon Nanotube Structure

Depending on the structure of the CNT they may or may not be electrically conductive. For example in the diagram above the armchair form is electrically conductive, but the others are semiconducting or metallically conducting depending on the orientation of the graphene sheet. The Chiral Vector, (n,m) of the zigzag form is (n,0) and the form is only metallically conductive if n is a multiple of 3. The chiral, (n,m) where m0, is only conductive if (2n+m)/3 is an integer.
While Carbon Nanotubes seem highly promising these are only their hypothetical values, in reality the results have been much different. Currently the largest CNT formed was less than a meter long and the properties are not always as expected. A lot more research is needed before these become a viable material for large structures, let alone a Space Elevator. Another disadvantage of Carbon Nanotubes is the fact that if there is a small fracture in their structure it can heavily reduce the tensile strength and even cause a Failure Cascade scenario, resulting in the breakage of the entire CNT string.

Summary

The Space Elevator is a very ambitious design concept, but we do not currently have the materials to make it. More research is needed into nanotechnology such as CNTs before they can become viable in their properties, size and quantity.
If the Elevator were to be constructed there would be radiation protection on the Climber, and the trip to the top of the elevator would take over two weeks so that the trip does not destabilise the Elevator with an excess of coriolis forces. I have created a simulation of most of the elevator using VPython. It shows the effects of the radiation belts, plots trajectories of projectiles released off the elevator and demonstrates tensions on the elevator.